BOARD OF TECHNICAL EXAMINATION – KARNATAKA

SUBJECT: APPLIED MATHEMATICS – II

For II - Semester

DIPLOMA COURSES OF ALL BRANCHES

Contact hour per week: 04 **Contact hour per Semester: 64**

UNIT NO.	CHAPTER TITLE	CONTACT HR.
	DIFFERENTIATION CALCULUS	
1	LIMITS	**6**
2	DIFFERENTIATION	**16**
3	APPLICATIONS OF DIFFERENTIATION	**6**
	INTEGRAL CALCULUS	
4	INDEFINITE INTEGRATION	**14**
5	DEFINITE INTEGRATION	**4**
6	APPLICATIONS OF DEFINITE INTEGRATION	**2**
7	**DIFFERENTIAL EQUATIONS**	12
8	TESTS AND ASSIGNMENTS	**4**
	TOTAL HOUR	**64**

REFFERENCE BOOKS:

1. Applied Mathematics –II By W.R Neelakanta. Sapna Publications.
2. Applied Mathematics –II By Dr. D S Prakash S Chand Publications
3. Text Books of PUC-2 mathematics.
4. Applied Mathematics –II for Polytechnics- By different Authors.
5. Engineering Mathematics.

QUESTION PAPER BLUE PRINT FOR APPLIED MATHEMATICS – II

UNIT NO.	HOUR	Questions to be set (2 Marks) PART- A	Questions to be set (5 Marks) PART- B	Questions to be set (5 Marks) PART- C	Questions to be set (5 Marks) PART- D
1	6	2	2	-	-
2	16	4	6	-	-
3	6	2	2	-	-
4	14	4	-	6	-
5	4	3	-	1	-
6	2	-	-	1	-
7	12	-	-	-	6
TOTAL	64	15	10	08	06
Questions to be answered		10	07	05	04

APPLIED – MATHEMATICS – II

	Marks	No. of Questions to be set	No. of Questions to be Answered
Part A	2 X 10 = 20	15	10
Part B	5 X 07 = 35	10	07
Part C	5 X 05 = 25	08	05
Part D	5 X 04 = 20	06	04
	Total = 100		

Diploma Courses of All Engineering Branches

II Semester

Sub: Applied Mathematics II

CONTENTS

UNIT – 1: LIMITS. **6 Hr.**

Variables and Constants. Definition of function. Types of function: Direct and Inverse functions, Explicit and implicit function, Odd and even functions (Definition with examples). Concept of x tends to 'a'. Definition of limit of a function. Problems on limit of a function by factorization, rationalization when x tend to '0', when x tend to '∞' and x tend to 'a'. Derivations of algebraic and trigonometric limits. Problems. Standard limit (only statement)

$$1.\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a \qquad 2.\lim_{x\to 0}\frac{e^x-1}{x}=1 \qquad 3.\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e \qquad 4.\lim_{n\to 0}(1+n)^{\frac{1}{n}}=e$$

Simple problems on standard limits.

UNIT – 2: DIFFERENTIAL CALCULUS. **16 Hr.**

Definition of increment and increment ratio. Definition of derivative of a function. Derivatives of functions of x^n, sinx, cosx and tanx with respect to 'x' from first principle method. List of standard derivatives. Rules of differentiation: Sum, product and quotient of functions. Problems on rules. Derivatives of function of a function (Chain rule). Derivatives of inverse Trigonometric functions, Hyperbolic functions and inverse of hyperbolic functions, Implicit functions, Parametric functions. Problems. Logarithmic differentiation. Problems. Successive differentiation up to second order. Problems.

UNIT – 3: APPLICATIONS OF DIFFERENTIATION. **6 Hr.**

Geometrical meaning of derivative. Equation of tangent and normal to the curve y = f(x) at a given point. Derivative as a rate measure. Definition of increasing and decreasing function. Maxima and minima of a function.

UNIT – 4: INTEGRAL CALCULUS. **14 Hr.**

Definition of Integration. List of standard integrals. Rules of integration (only statement)

$$1.\int kf(x)dx = k\int f(x)dx. \qquad 2.\int \{\mathrm{f(x)} \pm \mathrm{g(x)}\}\mathrm{dx} = \int \mathrm{f(x)dx} \pm \int \mathrm{g(x)dx}$$

problems. Integration by substitution method. Problems. Integrals of functions involving $a^2 + x^2$, $a^2 - x^2$ and their radicals. Some important integrals of the type

1.$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$ 2. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$ with proof.

3.$\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+c$ 4. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$ if $x \rangle a \rangle 0$.

5. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$ if $a \rangle x \rangle 0$. 6.$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+c$

7. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+c$ (3 to 7 no proof)

Integrals of the forms: $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \frac{px+q}{ax^2+bx+c}$dx , $\int \frac{px+q}{\sqrt{ax^2+bx+c}}$dx. Problems.

Integration by parts. Rule of integration by parts. Problems.

Integration of the forms: $\int e^x \left(f(x)+f^1(x)\right)dx$ Problems.

UNIT – 5: DEFINITE INTEGRALS. 4 Hr.

Definition of Definite integral. Theorems on definite integrals. Problems. Definite integrals of the type $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x}dx$, $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}dx$ Problems.

UNIT – 6: APPLICATIONS OF DEFINITE INTEGRALS. 2 Hr.

Find area, volume and r m s value of a function. Problems.

UNIT – 7: DIFFERENTIAL EQUATIONS. 12 Hr.

Definition, example, order and degree of differential equation with examples. Formation of differential equation by eliminating arbitrary constants up to second order. Solution of D E of first degree and first order by variable separable method. Solution of differential equations reducible to variable separable form. Linear equations and its solution. Solution of differential equations reducible to linear form-Bernoulli's form. Homogeneous form and its solution. Solution of differential equations reducible to homogeneous form. Exact differential equation and its solution. Solutions of differential equation of a type $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$, where a, b, c are constants. Simple problems.

GENERAL AND SPECIFIC OBJECTIVES

UNIT – 1: LIMITS. 6 Hr.

GENERAL OBJECTIVES.

1.1 To understand variables, constants and functions.

1.2 To know the indeterminant form and evaluation of limit of a given function.

SPECIFIC OBJECTIVES.

1.1.1 Explain calculus as calculation of Infinitesimal values.
1.1.2 Define Independent, dependent variable and constants with examples.
1.1.3 Define a function.
1.1.4 List types of functions.
1.1.5 Define 1. Direct and inverse functions 2. Explicit and Implicit function 3.Odd and even function with examples.
1.2.1 Explain the concept of x tends 'a'.
1.2.2 Define limit of a function.
1.2.3 Solve problems on limit of a function by factorization.
1.2.4 Solve problems on limit of a function by rationalization.
1.2.5 Solve problems on limit of a function when x tends INFINITY.

1.2.6 Deduce $\lim_{x\to a}\frac{x^n - a^n}{x-a} = na^{n-1}$ for any rational number.

$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ where θ is in radians

1.2.7 Solve problems on above results.

1.2.8. Write formula for standard limits(statement only).

$$1.\lim_{x\to 0}\left(\frac{a^x - 1}{x}\right) = \log_e a \quad 2.\lim_{x\to 0}\left(\frac{e^x - 1}{x}\right) = 1 \quad 3.\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e = \lim_{n\to 0}(1+n)^{\frac{1}{n}}$$

1.2.9 Solve simple problems on above results.

UNIT – 2: DIFFERENTIAL CALCULUS. 16 Hr.

GENERAL OBJECTIVES.

2.1 To understand the differentiation of a function in terms of limit of a function.

2.2 To know different methods of differentiation.

2.3 To know the derivatives of higher order up to second order.

SPECIFIC OBJECTIVES.

2.1.1 Define increment and increment ratio.

2.1.2 Define differentiation.

2.1.3 Derive the differential co-efficient (dy/dx).

2.1.4 Derive differentiation of a functions from first principle method x^n, Sinx, Cosx and Tanx with respect to 'x'.

2.1.5 State derivative of e^x and log x.

2.2.1 State rules of differentiation: Derivatives of Sum, Product and quotient of function.

2.2.2 Solve problems on rules of differentiation.

2.2.3 Obtain the derivatives of function of a function (Chain Rule), Inverse T-functions, Implicit functions & Parametric functions

2.2.4 Solve problems on above types.

2.2.5 Carry out logarithmic Differentiation

2.2.6 Solve problem of the type x^x, a^x, $x^{\text{Sin x}}$, $x^{1/x}$ etc.,

2.3.1 Obtain the second derivative of a function.

2.3.2 Solve problems on Successive differentiation.

UNIT -3: APPLICATIONS OF DIFFERENTIATION. 6Hr.

GENERAL OBJECTIVES.

3.1 To understand dy/dx as slope of a tangent.

3.2 To illustrate dy / dx as a rate measure.

3.3 To understand maxima and minima of a function.

SPECIFIC OBJECTIVES.

3.1.1 Explain geometrical meaning dy / dx as a slope of tangent.

3.1.2 Find equation of tangent and normal to a curve y = f(x) at a given point.

3.1.3 Solve problems on tangent and normals.

3.2.1 Explain derivative as a rate measure.

3.2.2 Obtain velocity and acceleration for a moving body whose equation of motion is given.

3.2.3 Solve problems on rate measure including variation of area, volume etc.,

3.3.1 Define increasing and decreasing function.

3.3.2 State the condition for maxima and minima of a function, no proof.

3.3.3 Find maximum and minimum values of a function.

UNIT – 4: INTEGRAL CALCULUS. 14 Hr.

GENERAL OBJECTIVES.

4.1 To know integration as converse process of differentiation.

4.2 To understand indefinite integral.

SPECIFIC OBJECTIVES.

4.1.1 Define integration as anti derivative.

4.1.2 List of standard integrals.

4.1.3 State rules of Integration.
4.1.4 Solve the problems on rules of integration.
4.2.1 Explain integration by substitution method.
4.2.2 Solve problems on substitution method.
4.2.3 Derive $1.\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$ $\quad 2.\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$.

4.2.4 Write

$3.\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+c \qquad 4.\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c \quad \text{if } x \rangle a \rangle 0.$

$5.\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c \quad \text{if } a \rangle x \rangle 0. \quad 6.\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+c$

$7.\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+c \qquad (\text{3 to 7 no proof})$

4.2.4 Solve problems on above results.
4.2.5 Write

$\int \frac{dx}{ax^2+bx+c} \;, \int \frac{dx}{\sqrt{ax^2+bx+c}} \;, \int \frac{px+q}{ax^2+bx+c}\,\text{dx} \;, \int \frac{px+q}{\sqrt{ax^2+bx+c}}\,\text{dx}.$

4.2.5 Solve problems on above results.
4.2.6 Explain the rule integration by parts.
4.2.7 Solve problems of the type x sinx, x^2 Cos x, (ax + b) e^x, x $\sin^2$x, log x, e^x sinx, x log x etc.,
4.2.8 Solve problems of the type $\int e^x \left(\text{f}(x) + \text{f}^1(x)\right)\text{dx}$

UNIT – 5: DEFINITE INTEGRALS. 4 Hr.

GENERAL OBJECTIVES.

5.1 To understand the concept of definite integral to eliminate constant of integration.

SPECIFIC OBJECTIVES.

5.1.1 State, $\int_a^b$ f(x) dx as a definite integral.
5.1.2 State theorems on definite integrals.
5.1.3 Solve the problems of the same type as in indefinite integral using limits of integration.
5.1.4 Solve definite integrals of the type $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x}dx, \quad \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}dx$

UNIT – 6: APPLICATIONS OF DEFINITE INTEGRALS. 2 Hr.

GENERAL OBJECTIVES.

6.1 To understand definite integral as a tool to find area under the curve, volume of solid of revolution and r m s value of a function.

SPECIFIC OBJECTIVES.

6.1.1 Explain definite integral as a limit of sum (statement).
6.1.2 Write the formulae for finding area, volume and r m s value of a function.
6.1.3 Solve problems on above applications.

UNIT – 7: DIFFERENTIAL EQUATIONS. 12 Hr.

GENERAL OBJECTIVES.

7.1 To understand the concept of differential equation.

7.2 To solve differential equation for unknown functions.

SPECIFIC OBJECTIVES.

7.1.1 Define differential equation with examples.
7.1.2 Define Order and Degree of D E with examples.
7.1.3 Formation of D E by eliminating arbitrary constants.
7.2.1 Obtain the solution of D E by variable separable method.

7.2.2 Solve problems on variable separable method.
7.2.3 Obtain the solution of D E by Reducible to variable separable method – Solve problems.
7.2.4 Obtain the solution of linear D E of the type $\frac{dy}{dx} + Py = Q$ --solve problems.
7.2.5 Obtain the solution of D E by Reducible to linear form (Bernoulli's form) –solve problems.
7.2.6 Obtain the solution of D E by Homogeneous form – solve problems.
7.2.7 Obtain the solution of D E by Reducible to homogeneous form – solve problems
7.2.8 Obtain the solution of Exact D E --- solve problems.
7.2.9 Obtain the solution of D E of the type $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0,$ -- Solve simple problems only.

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BOARD OF TECHNICAL EXAMINATION – KARANATAKA

MODEL QUESTION PAPER

Code:

APPLED MATHEMATICS –II (FOR ALL COURSES)

Time: 3 Hrs

Maximum marks:100

NOTE: i) Answer any 10 questions in section A, 7 questions in section B 5 questions in section C, & 4 questions in D

ii) Each question carries 2 marks in section A

iii) Each question carries 5 marks in remaining section

SECTION – A

1. Evaluate $\lim_{x \to 0} \frac{\sin^2 3x}{\tan^2 4x}$

2.Evaluate $\lim_{x \to 2} \frac{x^3 - 8}{x^4 - 16}$

3. Find $\frac{dy}{dx}$ if y = 1 –cos4x

4. Find $\frac{dy}{dx}$ if $x^2 + y^2 = a^2$

5. Find $\frac{dy}{dx}$ if x = a sinθ and y = a cosθ

6. Find the slope of a tangent to the curve $y = x^2+6x - 7$ at point (1, –2)

7. If $s = 4t^2 - 4t + 6$ then find velocity when t = 2second

8. Evaluate $\int e^x \tan e^x \, dx$

9. Evaluate $\int \sin^2 x \, dx$

10 Evaluate $\int \operatorname{cosec}(1-4x)dx$

11. Evaluate $\int_0^4 \frac{1}{\sqrt{x}} dx$

12. Evaluate $\int_0^{\frac{\pi}{2}} \sin x\, dx$

13. Evaluate $\int_0^{\frac{\pi}{2}} \tan^2 x\, dx$

14. Evaluate $\int \frac{\sin x}{\cos^2 x} dx$

15. Show that $\frac{d(a^x)}{dx}$ = a^x $\log_e a$

SECTION – B

1. Prove geometrically $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$

2. Evaluate $\lim_{x \to 0} \frac{3\sin 2x - 5x}{4x - \tan x}$ $\left(\frac{dy}{dx} + 1 \right) = e^x$

3. Find the derivative of tanx w.r.to x form the first principle

4. If y = e^x (sinx – cosx), find dy/dx.

5. If y = $\frac{2 + 3\sinh x}{3 + 2\cosh x}$, find dy/dx

6. If xcoy + ysin(x/y) = k find dy/dx.

7. Find dy/dx if x = a(cosθ + θ) and y = a (1 – sinθ)

8. If y = sinx $^{\cos x}$, find dy/dx.

9. Find the equations of tangent and normal to the curve y^2 = 9x at (1, - 3)

10. Find the maximum and minimum value of the fuction $2x^3$ - 12 x^2 + 18x + 5.

SECTION – C

1. Evaluate $\int \sin^3 x\, dx$

2. Evaluate $\int \frac{dx}{x^2 - 6x + 13}$

3. Evaluate $\int x^2 \sin x\, dx$

4. Evaluate $\int \frac{2x+3}{\sqrt{8 - 2x - x^2}} dx$

5. Evaluate $\int_0^{\frac{\pi}{2}} \cos^3 x\, dx$

6. Evaluate $\int x.\log x\, dx$

7. Evaluate $\int \frac{x^4}{x^2+1}\, dx$

8. find the area bounded by the curve $y = 4x - x^2 - 3$ and x-axis.

SECTION – D

Solve the following equations;

1. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$
2. $y(1 + x)\, dx + x(1 + y)\, dy = 0$
3. $(y^3 - 3x^2 y)\, dx - (x^3 - 3xy^2)\, dy = 0$
4. $(3y - 7x + 7)\, dx + (7y - 3x + 3)\, dy = 0$
5. $(2x + y + 1)\, dx + (x + 2y + 1)\, dy = 0$
6. $e^y \left(\frac{dy}{dx} + 1\right) = e^x$

0-o-0-o-0-o-0-o-0-o-0-o-0-o-0-o-0-o-0-o-0